(3.10), and therefore, the infinite systems of linear equations (3.1), (3.2) are quasi-completely regular.

BIBLIOGRAPHY

1. Arutiunian, N. Kh., Contact problem for a half-plane with elastic reinforcement. PMM Vol. 32, №4, 1968.
2. Shtaerman, I. Ia., Contact Problem of Elasticity Theory. Moscow-Leningrad, Gostekhizdat, 1949.
3. Chibrikova, L. I., On the solution of some complete singular integral equations. Uch. Zap. Kazan. Univ., Vol. 122, Book 3, 1962.
4. Kogan, Kh. M., On a singular integro-differential equation. Differ. Uravn., Vol. 3, №2, 1967.
5. Muskhelishvili, N. I., Some Fundamental Problems of Mathematical Elasticity Theory. 5th Ed., Moscow, "Nauka", 1966.
6. Gradshtein, I. S. and Ryzhik, I. M., Tables of Integrals, Sums, Series and Products. 4th Ed., Moscow, Fizmatgiz, 1962.
7. Szego, G., Orthogonal Polynomials. Moscow, Fizmatgiz, 1962.
8. Jahnke, E., Emde, F. and Loesch, F., Special Functions. Moscow, "Nauka", 1968.
9. Kantorovich, L. V. and Krylov, V. I., Approximate Methods of Higher Analysis, 5th Ed., Moscow-Leningrad, Fizmatgiz, 1962.
10. Bari, N. K., Trigonometric Series. Moscow, Fizmatgiz, 1961.
11. Liusternik, L. A. and Sobolev, V. I., Elements of Functional Analysis, 2nd. Ed., Moscow, "Nauka", 1965.

Translated by M. D. F.

# DUAL TRIGONOMETRIC SERIES IN CRACK AND PUNCH PROBLEMS

PMM Vol. 33, №5, 1969, pp. 844-849
B. A. KUDRIAVTSEV and V. Z. PARTON
(Moscow)
(Received December 19, 1968)

The author obtains the solution of a certain class of dual trigonometric series with the aid of a method proposed by Tranter [1]. Certain crack and punch problems, both static and dynamic, reduce to this class. As an example the problem of steady-state vibration of an unbounded plane with a periodic system of slits along the real axis is considered. The solution which is obtained permits the determination of a purely inertial effect which lowers the fracture load.

1. Let us consider the dual trigonometric series

$$\sum_{n=1}^{\infty} nB_n^* \cos n\xi = f(\xi) \qquad (0 \leqslant \xi \leqslant \xi_0)$$
$$\sum_{n=1}^{\infty} B_n^* \cos n\xi = 0 \qquad (\xi_0 \leqslant \xi \leqslant \pi) \tag{1.1}$$

Following Tranter [1], we represent the coefficients $B_n{}^*$ in the form

$$B_n{}^* = \frac{2}{\pi}\int_0^{\xi_0} V(\xi)\cos n\xi d\xi \tag{1.2}$$

$$V(\xi) = \cos{}^1/_2\,\xi \int_x^1 \frac{\chi(s)\,ds}{(s^2 - \sin^2 0.5\xi\,\csc 0.5\xi_0)^{1/2}}, \quad x = \sin{}^1/_2\,\xi\csc{}^1/_2\,\xi_0 \tag{1.3}$$

Substituting (1.3) into (1.2) and interchanging the order of integration, we obtain

$$B_n{}^* = 2\sin{}^1/_2\,\xi_0 \int_0^1 F(n,\ -n;\ 1;\ s^2\sin^2 0.5\,\xi_0)\,\chi(s)\,ds \tag{1.4}$$

where $F\ (n,\ -n;\ 1;\ s^2\sin^2 0.5\xi_0)$ is the hypergeometric function.

We integrate the first of Eqs. (1.1) from 0 to $\xi$, multiply the result by $\sin{}^1/_2\xi$ and integrate once more over the same interval. This gives us

$$\sum_{n=1}^{\infty} B_n{}^* \left[\frac{\sin{}^1/_2(2n-1)\,\xi}{2n-1} - \frac{\sin{}^1/_2(2n+1)\,\xi}{2n+1}\right] = I_c(\rho) \tag{1.5}$$

where

$$^1/_2\xi = \arcsin(\rho\sin{}^1/_2\,\xi_0), \qquad I_c(\rho) = \int_0^{\xi} G(\xi)\sin{}^1/_2\,\xi d\xi, \qquad G(\xi) = \int_0^{\xi} f(\xi)\,d\xi$$

Replacing the expression in square brackets in (1.5) by the corresponding hypergeometric series [2] and substituting (1.4) into (1.5), we obtain an equation for the determination of $\chi\ (s)$

$$\int_0^1 \chi(s)\sum\nolimits_c(s,\ \rho,\ \xi_0)\,ds = \frac{3I_c\ (\rho)}{8\rho^3\sin^4 0.5\xi_0} \qquad (0<\rho<1) \tag{1.6}$$

where

$$S_1 = \sum\nolimits_c(s,\rho,\xi_0) = \tag{1.7}$$

$$= \sum_{n=1}^{\infty} nF\,(n, -n;\,1;\,s^2\sin^2 0.5\xi_0)\,F\,(1+n,\,1-n;\,{}^5/_2;\,\rho^2\sin^2 0.5\xi_0)$$

This series can be summed [1]

$$S_1 = 0 \quad (\rho < s), \qquad S_1 = \frac{3\,(\rho^2 - s^2)^{1/2}}{4\rho^3\sin^2 0.5\xi_0} \quad (\rho > s) \tag{1.8}$$

After substituting (1.8) into (1.6), we obtain

$$\int_0^s \chi(s)\,(\rho^2 - s^2)^{1/2}\,ds = \frac{1}{2\sin^2 0.5\xi_0}\,I_c(\rho) \tag{1.9}$$

It is easy to show that the solution of (1.9) has the form

$$\chi(s) = \frac{2}{\pi}\frac{1}{2\sin^2 0.5\xi_0}\frac{d}{ds}\int_0^s \frac{I_c{}'(\rho)\,d\rho}{(s^2-\rho^2)^{1/2}} = \frac{2}{\pi}\frac{d}{ds}\int_0^s \frac{\rho G(\xi)\,d\rho}{\sqrt{(s^2-\rho^2)(1-\rho^2\sin^2 0.5\xi_0)}} \tag{1.10}$$

which, together with Eqs. (1.2) and (1.3), gives the solution in final form.

In applications it is useful to find the asymptotic expression for the series as $\xi \to \xi_0 + 0$. Differentiating Eq. (1.8) twice with respect to $\xi$ and taking into account the relation [2]

$$\frac{\sin {}^1/_2 (2n-1)\xi}{2n-1} - \frac{\sin {}^1/_2 (2n+1)\xi}{2n+1} = \frac{4}{3}\sin^3 0.5\xi n F\,(1+n,\ 1-n;\ {}^5/_2;\ \sin^2 0.5\,\xi)$$

we find

$$S_2 = \sum_{n=1}^{\infty} nF\,(n,\ -n;\ 1;\ s^2 \sin^2 0.5\,\xi_0) \cos n\xi = 0, \quad \sin {}^1/_2\xi < s \sin {}^1/_2\xi_0$$

$$S_2 = -\frac{\sin {}^1/_2\xi\,(1 - s^2 \sin^2 0.5\xi_0)}{4\,(\sin^2 0.5\xi - s^2 \sin^2 0.5\xi_0)^{3/2}}, \quad \sin {}^1/_2\xi > s \sin {}^1/_2\xi_0 \qquad (1.11)$$

We now multiply Eq. (1.11) by $2\sin {}^1/_2\ \xi_0 \quad \chi(s)$ and integrate from zero to one. Then, taking (1.1) and (1.4) into consideration, we obtain

$$S_3 = \sum_{n=1}^{\infty} nB_n{}^* \cos n\,\xi = f(\xi) \qquad (\xi < \xi_0)$$

$$S_3 = -{}^1/_2 \sin {}^1/_2\xi_0 \sin {}^1/_2\,\xi \int_0^1 \frac{(1 - s^2 \sin^2 0.5\xi_0)\,\chi(s)\,ds}{(\sin^2 0.5\xi - s^2 \sin^2 0.5\xi_0)^{3/2}} \qquad (\xi > \xi_0) \quad (1.12)$$

By carrying out an integration by parts it can be shown that (1.12) has the following singularity as $\xi \to \xi_0 + 0$:

$$S_3 = -{}^1/_2 \sin {}^1/_2\xi \operatorname{ctg} {}^1/_2\xi_0 \cos {}^1/_2\xi_0 \frac{\chi(1)}{(\sin^2 0.5\xi - \sin^2 0.5\xi_0)^{1/2}} + \ldots \quad (\xi > \xi_0) \ (1.13)$$

where all terms which are bounded as $\xi \to \xi_0$ are omitted.

**2.** Let us consider the problem of steady-state vibration of an unbounded plane having a periodic system of slits of length $2l$ along the real axis (the distance between the centers of the slits is $2L$). The normal traction $p = q\cos\omega t$ is applied to the edges of the slits.

The amplitudes of the displacements and stresses can be represented in the following form [3]:

$$u = \frac{\partial\varphi}{\partial x} + \frac{\partial\psi}{\partial y}, \qquad v = \frac{\partial\varphi}{\partial y} - \frac{\partial\psi}{\partial x} \qquad (2.1)$$

$$\sigma_x = 2\mu \frac{\partial}{\partial x}\left(\frac{\partial\varphi}{\partial x} + \frac{\partial\psi}{\partial y}\right) - \lambda \frac{\omega^2}{c_1^2}\varphi,\ \sigma_y = -2\mu \frac{\partial}{\partial x}\left(\frac{\partial\varphi}{\partial x} + \frac{\partial\psi}{\partial y}\right) - (\lambda + 2\mu)\frac{\omega^2}{c_1^2}\varphi$$

$$\tau_{xy} = 2\mu \frac{\partial}{\partial x}\left(\frac{\partial\varphi}{\partial y} - \frac{\partial\psi}{\partial x}\right) - \mu \frac{\omega^2}{c_2^2}\psi \quad c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad c_2^2 = \frac{\mu}{\rho} \qquad (2.2)$$

where $\lambda$ and $\mu$ are the Lamé constants and $c_1$ and $c_2$ are the longitudinal and transverse wave speeds. The functions $\varphi$ and $\psi$ are solutions of the equations

$$\nabla^2\varphi + \frac{\omega^2}{c_1^2}\varphi = 0, \quad \nabla^2\psi + \frac{\omega^2}{c_2^2}\psi = 0, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \qquad (2.3)$$

and, taking account of the behavior at infinity and the symmetry of the state of stress, they can be represented in the form

$$\varphi(x, y) = \sum_{n=1}^{\infty} A_n e^{-\alpha_n \Omega_1 y} \cos\alpha_n x, \quad \psi(x, y) = \sum_{n=1}^{\infty} B_n e^{-\alpha_n \Omega_2 y} \sin\alpha_n x \quad (2.4)$$

$$\alpha_n = \frac{\pi n}{L}, \quad \omega_1 = \frac{\omega}{\alpha_n c_1}, \quad \omega_2 = \frac{\omega}{\alpha_n c_2}, \quad \Omega_1 = \sqrt{1 - \omega_1^2}, \quad \Omega_2 = \sqrt{1 - \omega_2^2}$$

The coefficients $A_n$ and $B_n$ in the expressions (2.4) are to be determined from the following conditions:

$$\tau_{xy}(x,0)=0\ (|x|<\infty),\qquad \sigma_y(x,0)=-q\qquad (0\leqslant x\leqslant l)$$
$$v(x,0)=0\qquad (l\leqslant x\leqslant L)\tag{2.5}$$

We find from (2.5) that

$$A_n=-B_n\frac{1-{}^1/_2\,\omega_2^2}{\Omega_1}$$

$$2\mu\sum_{n=1}^{\infty}B_n\Big[\Omega_2-\frac{(1-{}^1/_2\,\omega_2^2)^2}{\Omega_1}\Big]\alpha_n^2\cos\alpha_n x=-q\quad (0\leqslant x\leqslant l)\tag{2.6}$$

$$\frac{1}{2}\sum_{n=1}^{\infty}\alpha_n\omega_2B_n\cos\alpha_n x=0\quad (l\leqslant x\leqslant L)\tag{2.7}$$

In the determination of the coefficients $B_n$ satisfying the dual trigonometric series, we shall assume $\omega_1^2=\dfrac{\omega^2}{\alpha_n^2c_1^2}<1,\quad \omega_2^2=\dfrac{\omega^2}{\alpha_n^2c_2^2}<1\qquad (n=1,2,\ldots)$

Thus the following expressions are valid:

$$\Omega_1=\sum_{k=0}^{\infty}d_k\omega_1^{2k},\quad \Omega_2=\sum_{k=0}^{\infty}d_k\omega_2^{2k},\quad \Omega_2-\frac{(1-{}^1/_2\,\omega_2^2)^2}{\Omega_1}=-\sum_{k=1}^{\infty}D_k\omega_1^{2k}\tag{2.8}$$

where $d_0=1,\ d_1={}^1/_2,\ldots$

$$D_k=d_k\Big(\frac{c_1^2}{c_2^2}\Big)^k+2(k+1)d_{k+1}-2kd_k\Big(\frac{c_1^2}{c_2^2}\Big)+\frac{1}{2}(k-1)d_{k-1}\Big(\frac{c_1^2}{c_2^2}\Big)^2$$

Considering (2.8), we may rewrite Eqs. (2.6) and (2.7) as:

$$\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}D_k\omega_1^{2k-2}\alpha_nB_n^*\cos\alpha_n x=\frac{q}{2\mu}\frac{c_1^2}{\omega^2}\qquad (0\leqslant x\leqslant l)$$
$$\sum_{n=1}^{\infty}B_n^*\cos\alpha_n x=0\qquad \Big(B_n^*=\frac{B_n}{\alpha_n}\Big)\qquad (l\leqslant x\leqslant L)\tag{2.9}$$

We shall seek the coefficients $B_n^*$ in the form of the series.

$$B_n^*=B_{n,-1}^*\Big(\frac{c_1^2}{\omega^2}\Big)+B_{n,0}^*+B_{n,1}^*\Big(\frac{\omega^2}{c_1^2}\Big)+B_{n,2}^*\Big(\frac{\omega^2}{c_1^2}\Big)^2+\ldots\tag{2.10}$$

Substituting (2.10) into (2.9) and comparing coefficients of the same powers of $\omega^2/c_1^2$, we obtain the following sequence of dual trigonometric series for finding $B_{n,-1}^*$, $B_{n,0}^*$, $B_{n,1}^*$, ...:

$$\sum_{n=1}^{\infty}nB_{n,-1}^*\cos n\xi=\frac{L}{\pi}\frac{q}{2\mu D_1}\qquad (0\leqslant\xi\leqslant\xi_0)$$
$$\sum_{n=1}^{\infty}B_{n,-1}^*\cos n\xi=0\qquad (\xi_0\leqslant\xi\leqslant\pi)\tag{2.11}$$

$$\sum_{n=1}^{\infty}nB_{n,0}^*\cos n\xi=-\frac{D_2}{D_1}\Big(\frac{L}{\pi}\Big)^2\sum_{n=1}^{\infty}\frac{B_{n,-1}^*}{n}\cos n\xi\qquad (0\leqslant\xi\leqslant\xi_0)$$
$$\sum_{n=1}^{\infty}B_{n,0}^*\cos n\xi=0\qquad (\xi_0\leqslant\xi\leqslant\pi)\tag{2.12}$$

$$\sum_{n=1}^{\infty} nB_{n,1}^{*}\cos n\xi = -\frac{D_2}{D_1}\left(\frac{L}{\pi}\right)^2 \sum_{n=1}^{\infty} \frac{B_{n,0}^{*}}{n}\cos n\xi - $$

$$-\frac{D_3}{D_1}\left(\frac{L}{\pi}\right)^4 \sum_{n=1}^{\infty} \frac{B_{n,-1}^{*}}{n^3}\cos n\xi \qquad (0 \leqslant \xi \leqslant \xi_0)$$

$$\sum_{n=1}^{\infty} B_{n,1}^{*}\cos n\xi = 0 \qquad (\xi_0 \leqslant \xi \leqslant \pi) \tag{2.13}$$

$$(\xi = \pi x / L, \ \xi_0 = \pi l / L)$$

In what follows it will be sufficient (as calculations indicate) to limit ourselves to three terms of the expansion (2.10). Taking account of this, the normal stresses on the axis between slits are determined by the following relation:

$$\frac{1}{2\mu}\sigma_y(x, 0) = -D_1\left(\frac{\pi}{L}\right)\sum_{n=1}^{\infty} nB_{n,-1}^{*}\cos n\xi - \left(\frac{\omega^2}{c_1^2}\right)\sum_{n=1}^{\infty}\left[\frac{L}{\pi}D_2\frac{B_{n,-1}^{*}}{n} + \right.$$

$$\left. + \frac{\pi}{L}D_1 B_{n,0}^{*} n\right]\cos n\xi - \left(\frac{\omega^2}{c_1^2}\right)^2 \sum_{n=1}^{\infty}\left[D_3\left(\frac{L}{\pi}\right)^3\frac{B_{n,-1}^{*}}{n^3} + \right. \tag{2.14}$$

$$\left. + D_2\left(\frac{L}{\pi}\right)\frac{B_{n,0}^{*}}{n} + D_1\left(\frac{\pi}{L}\right)B_{n,1}^{*} n\right]\cos n\xi$$

$$(\xi_0 \leqslant \xi \leqslant \pi)$$

Considering that series of the form

$$\sum_{n=1}^{\infty} \frac{B_{n,j}^{*}}{n^k}\cos n\xi \qquad (k = 1, 3, 5, \ldots)$$

are continuous functions for $\xi = \xi_0$, we obtain from (1.13), (2.14), and the condition of limiting equilibrium [4] (*)

$$K_c = \lim_{x\to l}\sqrt{2\pi(x-l)}\,\sigma_y(x, 0) = \mu D_1 \cos {}^1/_2\,\xi_0\left(\frac{L}{\pi}\operatorname{tg} {}^1/_2\,\xi_0\right)^{-1/2}\sum_{j=0}^{\infty}\left(\frac{\omega^2}{c_1^2}\right)^j \chi_{j-1}(1) \tag{2.15}$$

where $K_c$ is the fracture cohesiveness. Determining the $\chi_{j-1}(1)$ $(j = 0, 1, 2)$ sequentially from (1.10) and (2.11–2.13), we obtain

$$\chi_{-1}(1) = \left(\frac{L}{\pi}\right)\frac{q}{\mu D_1}\frac{\operatorname{tg} {}^1/_2\,\xi_0}{\cos {}^1/_2\,\xi_0} \tag{2.16}$$

$$\chi_0(1) = \left(\frac{L}{\pi}\right)^3\frac{q}{\mu D_1}\left(\frac{D_2}{D_1}\right)[N_{-1}^1(\xi_0) + M_0(1, \xi_0)]\frac{\operatorname{tg} {}^1/_2\,\xi_0}{\cos {}^1/_2\,\xi_0}$$

$$\chi_1(1) = \left(\frac{L}{\pi}\right)^5\frac{q}{\mu D_1}\left\{\left(\frac{D_2}{D_1}\right)^2[N_0^1(\xi_0) + M_1(1, \xi_0)] + \right.$$

$$\left. + \frac{D_3}{D_1}[N_{-1}^3(\xi_0) - N_{-1}^1(\xi_0) M_0(1, \xi_0)]\right\}\frac{\operatorname{tg} {}^1/_2\,\xi_0}{\cos {}^1/_2\,\xi_0}$$

*) The condition of limiting equilibrium holds also in the same form for the case of dynamic loading (as can be justified by considerations of invariance [5]). However, $K_c$ will depend on the frequency of vibration $\omega$ and is determined experimentally.

$$N_{-1}^1(\xi_0) = -4\sin^2 0.5\xi_0 \int_0^1 \left[\sum_{n=1}^{\infty} \frac{1}{n} F(n, -n; 1; s^2\sin^2 0.5\xi_0)\right] \frac{sds}{1 - s^2\sin^2 0.5\xi_0} =$$

$$= 4\sin^2 0.5\xi_0 \int_0^1 \ln(s \sin {}^1/_2\,\xi_0)\, \frac{sds}{1 - s^2\sin^2 0.5\,\xi_0}$$

$$N_{-1}^3(\xi_0) = -4\sin^2 0.5\xi_0 \int_0^1 \left[\sum_{n=1}^{\infty} \frac{1}{n^3} F(n, -n; 1; s^2\sin^2 0.5\,\xi_0)\right] \frac{sds}{1 - s^2\sin^2 0.5\xi_0}$$

$$M_0(s, \xi_0) = {}^1/_6\,\pi \frac{\cos^2 0.5\xi_0}{\sin {}^1/_2\,\xi_0} \frac{d}{ds} \int_0^s \frac{\rho\xi^3 d\rho}{\sqrt{(s^2 - \rho^2)(1 - \rho^2\sin^2 0.5\xi_0)}} \tag{2.17}$$

$$N_0^1(\xi_0) = [N_{-1}^1(\xi_0)]^2 + 4\,\mathrm{tg}^2 0.5\,\xi_0 \int_0^1 \ln(s\sin {}^1/_2\,\xi_0)\, M_0(s, \xi_0)\, ds$$

$$M_1(s, \xi_0) = {}^1/_{120}\,\pi \frac{\cos^2 0.5\xi_0}{\sin {}^1/_2\xi_0} \frac{d}{ds} \int_0^s \frac{\rho\xi^5 d\rho}{\sqrt{(s^2 - \rho^2)(1 - \rho^2\sin^2 0.5\xi_0)}}$$

Taking account of the three terms of the series (2.15), we obtain the final expression

$$\frac{K_c}{\sqrt{2\pi}} = q\left(\frac{L}{\pi}\,\mathrm{tg}\,{}^1/_2\xi_0\right)^{1/2}\left\{1 + \left(\frac{\omega L}{\pi c_1}\right)^2\left(\frac{D_2}{D_1}\right)[N_{-1}^1(\xi_0) + M_0(1, \xi_0)] + \right. \tag{2.18}$$

$$\left. + \left(\frac{\omega L}{\pi c_1}\right)^4\left[\left(\frac{D_2}{D_1}\right)^2(N_0^1(\xi_0) + M_1(1, \xi_0)) + \left(\frac{D_3}{D_1}\right)(N_{-1}^3(\xi_0) - N_{-1}^1(\xi_0)\, M_0(1, \xi_0))\right]\right\}$$

For $\omega = 0$ in (2.18) we obtain the known result [6] of the solution of the problem for a plane with a periodic system of cracks along the $x$-axis when a static normal loading is applied to the edges of the cracks, the loading $q$ being constant along a slit.

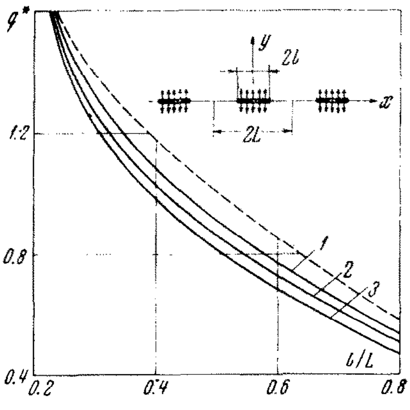


Fig. 1

Curves are shown in Fig. 1 for the relationship (2.18) for various frequencies of vibration of the external loading

$$q^* = \frac{q}{K_c}\sqrt{2L}$$

$$\left(\alpha = \frac{\omega L}{\pi c_1},\quad \nu = \frac{1}{3},\quad \frac{c_1^2}{c_2^2} = 4\right)$$

The dashed line corresponds to the static case $\omega = 0$. The curves 1, 2 and 3 correspond to the values $\alpha = 0.214$, $0.224$ and $0.316$. The solutions which have been constructed show that a purely inertial effect results in a decrease of the magnitude of the fracture load for a given crack length. This was noted in [7] for a single crack.

The relation (2.18) can also be used to determine the length of crack in a strip of length $2L$ for fracture to occur under the action of a normal load $p = q\cos\omega t$ applied to the edges of the crack.

BIBLIOGRAPHY


1. Tranter, C. L., Dual Trigonometrical Series. Proc. of the Glasgow Math. Assn., Vol. 4, pt. 2. p. 49, 1959
2. Gradshtein, I. S. and Ryzhik, I. M., Tables of Integrals, Sums, Series and Products. Moscow, Fizmatgiz, 1968.
3. Sneddon, I. N. and Berry, D. S., The Classical Theory of Elasticity. M., Fizmatgiz, 1961.
4. Irwin, G. R., Fracture. Handbuch der Physik, Berlin, Springer-Verlag, Vol. 6, 1958.
5. Cherepanov, G. P., Cracks in solids. Int. J. of Solids and Structures, Vol. 4, №8, 1968.
6. Barenblatt, G. I. and Cherepanov, G. P., On the effect of boundaries of a boby on crack development in brittle fracture. Izv. Akad. Nauk SSSR, OTN, (Eng. Sciences Div.). Mekhanika i mashinostroenie, №3, 1960.
7. Parton, V. Z. and Kudriavtsev, B. A., A dynamic problem for a plane with a slit. Dokl. Akad. Nauk SSSR, Vol. 185, №3, 1969.


Translated by A. R. R.

# HOMOGENEOUS SOLUTIONS OF TWO-DIMENSIONAL PROBLEMS OF THE THEORY OF ELASTICITY FOR A RECTANGULAR REGION OF A COSSERAT MEDIUM

PMM Vol. 33, №5, 1969, pp. 850-854

L. N. TER-MKRTICH'IAN
(Leningrad)

(Received October 18, 1968)

The theory of media whose properties are derived from the variational principle termed "euclidean action" by E. and F. Cosserat is developed in the book by Appell [1]. The arguments of the integrand in the mathematical expression of this principle are entirely determined by the geometry of the space. In the general statical case, 21 independent kinematical elements occur in this function for euclidean space. Certain contemporary authors, [2, 3] and others reduce the number of arguments to 14 by imposing differential relations among some of them. In particular, the components of the displacement vector and the components of the curl of the displacement vector are examined. Proceeding from the last precondition of [4], the basic apparatus for solution of the two-dimensional problem of the theory of elasticity is given and used in the present paper.

**1. The solution of the two-dimensional problem with the aid of the Airy and Mindlin stress functions.** The components of the stresses and couple stresses may be expressed with the aid of two stress functions in the following way:

$$\sigma_x = \frac{\partial^2 \varphi}{\partial x^2} - \frac{\partial^2 \psi}{\partial x \partial y}, \quad \mu_x = \frac{\partial \psi}{\partial x}, \quad \sigma_y = \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \psi}{\partial x \partial y}, \quad \mu_y = \frac{\partial \psi}{\partial y}$$

$$\tau_{xy} = -\frac{\partial^2 \varphi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y^2}, \quad \tau_{yx} = -\frac{\partial^2 \varphi}{\partial x \partial y} + \frac{\partial^2 \psi}{\partial x^2} \tag{1.1}$$

where the function $\varphi$ is the usual Airy stress function and $\psi$ is the stress function introduced by Mindlin.